

An Improved Fixed-Time Consensus Control for Linear Multi-agent Systems under Switching Topology with Link Failure

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ABSTRACT

In the consensus problem, the communication network describing the interactions between agents may change due to link failures and environmental conditions, which plays an important role in the stability and overall performance of the system. Also, faster convergence is always of interest in some practical engineering applications. For this purpose, the design of a fixed-time controller is of interest, because for this type of controller, the convergence time of the system states does not depend on the initial conditions of the agents. This paper investigates the problem of leaderless consensus control for second-order linear multi-agent systems (MASs) under a switching topology with link failures. To address this problem, we proposed an improved fixed-time distributed controller for second-order consensus in MASs with velocity and position coupling that guarantees convergence within a specified time interval. Compared to previous studies, the effects of simultaneous switching topology with link failures on second-order linear MASs are considered, so that the communication graphs are considered piecewise fixed and undirected jointly-connected ones. The stability of the proposed controller is proven based on graph theory, Lyapunov theorem and bi-limit homogeneity method. Finally, the effectiveness of the proposed method is demonstrated through numerical simulations, considering different periodicity for changes between topologies.

Keywords: Consensus, Fixed-time, Multi-agent Systems, Switching Topology, Link Failure

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کنترل اجماعی زمان-ثابت بهبودیافته برای سیستم‌های چندعاملی خطی تحت توپولوژی سوئیچینگ با خرابی لینک

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چکیده

در مسئله اجماع، شبکه ارتباطی که تعاملات بین عامل‌ها را توصیف می‌کند، ممکن است به دلیل خرابی لینک و شرایط محیطی تغییر کند که نقش مهمی در پایداری و عملکرد کلی سیستم ایفا می‌کند. همچنین، همگرایی سریع‌تر همیشه در برخی از کاربردهای مهندسی عملی مورد توجه است. برای این منظور، طراحی یک کنترل‌کننده زمان ثابت مورد توجه است، زیرا برای این نوع کنترل‌کننده، زمان همگرایی حالت‌های سیستم به شرایط اولیه عامل‌ها بستگی ندارد. این مقاله به بررسی مسئله کنترل اجماع بدون رهبر برای سیستم‌های چندعاملی خطی مرتبه دوم (MAS) تحت توپولوژی سوئیچینگ با خرابی لینک می‌پردازد. برای پرداختن به این مشکل، ما یک کنترل‌کننده توزیع‌شده زمان ثابت بهبود یافته برای اجماع مرتبه دوم در MAS‌ها با کوپلینگ سرعت و موقعیت پیشنهاد کردیم که همگرایی را در یک بازه زمانی مشخص تضمین می‌کند. در مقایسه با مطالعات قبلی، اثرات توپولوژی سوئیچینگ همزمان با خرابی لینک بر MAS‌های خطی مرتبه دوم در نظر گرفته شده است، به طوری که نمودارهای ارتباطی به صورت قطعه‌ای ثابت و غیرجهت‌دار متصل به هم در نظر گرفته می‌شوند. پایداری کنترل‌کننده پیشنهادی بر اساس نظریه گراف، قضیه لیاپانوف و روش همگنی دو حدی اثبات شده است. در نهایت، اثربخشی روش پیشنهادی از طریق شبیه‌سازی‌های عددی، با در نظر گرفتن تناوب متفاوت برای تغییرات بین توپولوژی‌ها، نشان داده می‌شود.

کلیدواژه‌ها: اجماع، زمان-ثابت، سیستم‌های چندعاملی، توپولوژی سوئیچینگ، خرابی لینک

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1- Introduction

Within the past two decades, MASs have grown into an important area of research because of their extensive applications in robotics, sensor networks, autonomous vehicles and power systems. These systems consist of multiple agents interacting by sharing their local information for mutual objective execution without any centralized control [1]. A classic problem in such a setting is the consensus problem, where one has to devise appropriate control protocols such that all agents agree on some key variables [2]. While most of the classical consensus algorithms achieve such an agreement asymptotically, there is a growing interest in fixed-time consensus where agents reach an agreement within a guaranteed time frame [3]. It is particularly effective in areas where fast, reliable results have to be achieved.

In practice, MASs are rarely static. Typically, agents operate in dynamic environments where the communication links between agents may vary with time. For instance, agents communication due to obstacles or temporary failure of network connections [4, 5]. The topologies would then be switching, which means that the interaction network between agents is changed. Changes like these introduce new difficulties when designing consensus algorithms, because they may cause delays in consensus or instability [6]. Various methods have been advanced to handle the issue of switching topology, but reaching consensus in a fixed time remains an open problem [7, 8].

With the emergence of the idea of using network communication in consensus control, related issues such as network changes and cyber-attacks in fixed-time consensus have been the subject of many studies in recent years. In [9], a finite-time consensus controller is proposed, using which the leader-follower consensus under a fixed directed and undirected communication network is implemented for a nonlinear second-order system by applying the Laplacian matrix. A robust distributed finite-time controller is proposed for the tracking problem of nonlinear high-order systems in the presence of external disturbances and model uncertainties under a fixed communication network, which is based on a terminal sliding mode controller that ensures that the states of the agents converge to the virtual leader [10]. In [11], a local finite-time consensus approach is presented for linear second-order systems under a fixed communication network with the help of the Lyapunov function and bi-limit homogeneity method. A distributed algorithm for the consensus of nonlinear second-order multi-agent systems in the presence of external disturbances and system uncertainties under an undirected communication network is presented, which ensures the convergence of the system in a fixed time [12].

Moreover, link failures caused by disruptions in communication or various types of attacks are relatively common [13]. Such link failures make the process of information exchange between agents difficult, further complicating the process of ensuring that all agents in the network can reach consensus. A significant challenge for multi-agent systems has been handling link failures while ensuring fixed-time consensus.

To deal with this problem, a fixed-time control approach is proposed for the nonlinear first-order system under switching topology, which finally admits an upper bound independent of the initial conditions of the agents [14]. Then the proposed algorithm, [14], was extended for linear second-order systems under fixed topology for leader-following case [15]. A fixed-time consensus method is presented by defining the sliding surface for nonlinear second-order systems under fixed topology, which has proved the stability of the proposed method by using a Lyapunov function and bi-homogeneous property [16], and then with a similar approach, the robust second-order consensus control and fixed-time fault-tolerant consensus control were developed for high-order nonlinear system [3, 13]. A fixed-time control approach has been developed for both leaderless and leader-follower cases in the presence of external disturbances, where the

settling time is bounded uniformly according to the initial conditions [17].

The effect of switching topology in fixed-time consensus is presented in [8], where the selection of the controller gain is also done using the design of a fixed-time output observer. Ref. [18] presents a control approach that makes the second-order linear system reach consensus in finite time under both fixed and switching topologies for leader-follower case. In [19], a finite-time controller for a linear first-order system is proposed, which guarantees the consensus of agents under a variable communication network with a Markov process. Two finite-time and fixed-time controllers are proposed for the consensus of linear first-order systems, which guarantee the convergence of the system to the average of the initial conditions under a variable connected communication network [20]. In [21], the finite-time adaptive tracking conditions for nonlinear second-order multi-agent systems with unknown parameters under a variable communication network are presented in the form of finite-time Lyapunov stability and the dwell time method.

On the other hand, the consensus problem of MASs under cyber-attacks has attracted much attention. However, a review of previous research shows that there are few results on the secure fixed-time controller for MASs under cyber-attacks. In [22], the problem of fixed-time consensus for MASs under denial-of-service (DoS) attacks is investigated, but in this work, attack detection algorithms are needed. A secure consensus controller in the presence of malicious agents is proposed, which is done by using an iterative learning algorithm to solve the problem of repelling attacks resulting from the disconnection and reconnection of communication links in a finite time [23]. In [13], a secure consensus controller is proposed, which guarantees convergence under a variable communication network and a DoS attack for general linear dynamics. In [24], a finite-time consensus controller for general linear dynamics under a DoS attack and a fixed communication network is also investigated, where all links are down and each agent is left alone. First, a time-varying dynamic model is proposed that describes the DoS attack in terms of attack frequency and duration. An intuitive relationship between attack parameters and feedback gain is also presented. An event-based consensus controller for a nonlinear first-order system under a DoS attack is presented, which guarantees reaching consensus in finite time under a fixed directed communication network [25]. In [26], a control algorithm is proposed that makes the linear system achieve a finite-time consensus under fixed topology with link failure, which is assumed that the link failure is done through a special case DoS attack. In this paper, an improved fixed-time consensus control is proposed for linear second-order MASs operating under switching topology and link failure. Communication topologies are considered as undirected jointly-connected and also, according to the above-mentioned papers, after each topology change, failure occurs for some links, which in this paper is caused by DoS attack. The aim of this approach is to provide a possibility for the agents to reach a consensus in a finite time, regardless of the initial conditions or the number of times the communication network changes. The effectiveness of this approach is demonstrated through numerical simulations and the effects of changing the topologies periodicity are also investigated using simulations. Compared to previous research, the main innovations of this paper are as follows:

- Designing an improved fixed-time consensus controller for second-order linear multi-agent systems that simultaneously considers two common network disturbances: (a) topology switching with average dwell time (b) communication link fails caused by DoS attacks.
- Considering a more realistic model of link failure, such that after each topology change, a DoS attack is probabilistically applied to the edges, but the overall graph connectivity is maintained at certain time intervals. This

model is more complex than similar works that have considered only one of these factors [8, 13, 18].

The rest of the paper is organized as follows: Section 2 presents the general concepts and the problem formulation; Section 3 proposes the design of a fixed-time consensus controller considering variable topology and link failures. The results of numerical simulations are shown in Section 4. Finally, the discussion and conclusion are presented in Section 5.

2- Problem Statement

2-1- Graph Theory

We look into a MAS which includes N agents, that have the capability for communication and interaction. In this section, we represent the structure of communication among agents using graph theory. The behavior of the agents is represented by an undirected graph $G(V, E, A)$, where $V = \{1, 2, \dots, N\}$ indicates the vertices (agents), E indicates the edges that represent the connected pairs of agents, and $A = [a_{ij}] \in R^{N \times N}$ is an adjacency matrix of the graph. The value a_{ij} equals 1 if and only if the edge $(j, i) \in E$, representing that agent j has the ability to send information to agent i . For the undirected graph G , the symmetric matrix A is considered, where $a_{ij} = a_{ji}$. From this type of graph, when $a_{ij} = a_{ji} > 0$, this implies that agent i is allowed to communicate with agent j both by sending and receiving simultaneously. The set of neighbors of each agent i is represented as $N_i = \{j | j \in V: (j, i) \in E\}$ [27].

2-2- Switching Topology

We consider a multi-agent system wherein the communication topology switches between a set of undirected graphs $\{G_1, G_2, \dots, G_l\}$. The switching signal is defined by $\sigma(t): [0, \infty) \rightarrow L = \{1, 2, \dots, l\}$, and $G_{\sigma(t)}$ denotes the communication graph that is active at time t . There is a strictly increasing sequence $\{\xi_m, m \geq 0\}$ such that for every $t \in [\xi_m, \xi_{m+1})$, $\sigma(t) = m$, where $m \in L$ [28].

Definition 1: The average dwell time, τ_a , for the switched topologies are bounded by:

$$N(0, t) \leq \frac{t}{\tau_a} \quad (1)$$

where $N(0, t)$ denotes the number of topology changes occurring in time interval $(0, t)$ [29].

2-3- DoS Attack

DoS attacks occur by impeding communication between neighboring agents, which in turn obstructs the flow of data among them. This paper deals with the fixed-time consensus in MASs for scenarios where some communication channels are made unavailable by DoS attacks.

Let a_k and A_k represent the start time and duration of the k -th DoS attack, with $A_0 = 0$. As shown in Fig. 1, the k -th attack interval is defined as $\mathcal{A}_k = [a_k, a_k + A_k)$. The "attack-free" time interval is defined as $\Xi(t) = [0, t] \setminus \Theta(t)$, where $\Theta(t) = \bigcup_{k \in N} \mathcal{A}_k$ represents the whole attack period in the interval $[0, t]$ [24].

The intensity of the DoS attacks is characterized by two factors: frequency and duration. We make the following assumptions regarding these parameters. Since attackers are always limited by accessible resources, these assumptions can be reasonable [13].

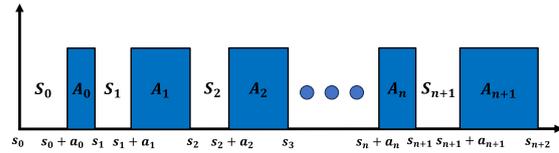


Fig. 1 DoS attack signal [13].

Assumption 1 (Frequency [30]): There are positive constants $\varepsilon_f \geq 0$ and τ_f such that for all $t \geq t_0$:

$$f(t_0, t) \leq \varepsilon_f + \frac{t - t_0}{\tau_f} \quad (2)$$

where $n(t_0, t)$ is the number of attacks falling in the interval $[t_0, t]$.

Assumption 2 (Duration [30]): There are exist positive constants $\varepsilon_d \geq 0$ and τ_d such that for all $t \geq t_0$:

$$[\Theta(t)] \leq \varepsilon_d + \frac{t - t_0}{\tau_d} \quad (3)$$

where $[\Theta(t)]$ is the total attack duration within the interval $[t_0, t]$.

2-4- Problem Formulation

The second-order linear dynamic model is chosen because it is widely used to describe the dynamic behavior of physical agents such as mobile robots and autonomous vehicles. This model provides a convenient simplification for the theoretical analysis of the main challenges of this paper, namely achieving fixed-time consensus under switching topologies with link failure. Furthermore, many nonlinear systems can be approximated around the equilibrium point by such a linear model. Assume that the agents follow a second-order linear dynamic model [15]:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases} \quad (4)$$

where $x_i \in R^n$ and $v_i \in R^n$ represent the state of each agent where, $i = 1, 2, \dots, N$. $u_i \in R^n$ is the acceleration level input that depends only on the states of agent i and its neighbors. In other words, agent i updates its states only based on information from its neighbors. Hence, u_i must be a local rule that can also be considered as a state feedback controller. The main task is to design a control law u_i for each agent such that consensus among all agents is achieved within a fixed time, for arbitrary any initial condition.

2-5- Useful Lemmas and Definitions

Let us consider the system described by the equation:

$$\dot{x} = f(x) \quad , \quad x(0) = x_0 \quad (5)$$

the nonlinear function $f: R^n \rightarrow R^n$ is defined such that $f(0) = 0$, implies that $x = 0$ is the equilibrium point (5). The standard definitions now follow.

Definition 1 (Global Finite-Time Stability): The equilibrium point of system (5) is said to be globally finite-time stable if it is globally asymptotically stable and there exists a settling-time function $T: R^n \rightarrow R_{\geq 0}$ such that for every initial condition $x_0 \in R^n$, the solution $x(t, x_0)$ satisfies $x(t, x_0) = 0$ for all $t \geq T(x_0)$ [12].

Definition 2 (Fixed-Time Stability): The equilibrium point of system (5) is said to be fixed-time stable if it is globally finite-time stable and the settling-time function $T(x_0)$ is bounded by some constant $T_{max} > 0$, i.e., $\sup_{x_0 \in R^n} T(x_0) \leq T_{max}$ [12].

Lemma 1: Assuming that the interaction graph G has a spanning tree or an undirected and connected graph, then the set of equilibrium points of the system (4) under the consensus protocol is equal to $S_e = \{k1 : k \in \mathbb{R}\}$ which is a linear spanning space $span(1)$ [31].

Lemma 2 (Non-smooth Lassalle invariance principle): Consider the system $\dot{x} = f(x)$, where $x(t)$ is a solution to the initial condition $x(0) = x_0 \in R^n$ and the function $f: U \rightarrow R^n$ is continuous on an open subset. Assume that $V(x): U \rightarrow R$ is a local Lipschitz function such that its derivative on paths is less than or equal to zero ($D^+V(x) \leq 0$). Then, the positive limit set $A^+(x_0) \cap U$ lies in the

union of all solutions remaining in the invariant set $S = \{x \in U: D^+V(x) = 0\}$ [32].

Lemma 3: The following system is considered:

$$\dot{x} = f(x) \quad (6)$$

where $x = [x_1, x_2, \dots, x_n]^T$ and $f(x) = [f_1(x), f_2(x), \dots, f_n(x)]^T$. If the system (4) has a degree of homogeneity σ_1 with the dilation (a_1, a_2, \dots, a_n) and a degree of homogeneity σ_2 with the dilation (b_1, b_2, \dots, b_n) and a continuous function f , then the equilibrium point is asymptotically stable. If $\sigma_1 < 0$ in the 0 -limit and $\sigma_2 > 0$ in the ∞ -limit, then the system is called homogeneous in bi-limit and when such a property is satisfied, the equilibrium point is globally fixed-time stable [33].

3- Controller Design

Let a multi-agent system be comprised of n nodes. Inter-agent communication is modeled by an undirected graph G , where an edge between agents i and j , denoted by a symmetric weight $(a_{ij} = a_{ji})$, establishes a bidirectional information channel. The neighborhood of an agent i is defined as $N_i(E)$. For a set of m graphs $\{G_1, G_2, \dots, G_m\}$ sharing the common vertex set V , the union graph G_{1-m} is constructed such that its edge set is the union of the edge sets of all constituent graphs. The weight of an edge in G_{1-m} is given by the summation of the corresponding edge weights from G_1 to G_m . The set $\{G_1, G_2, \dots, G_m\}$ is deemed jointly-connected if and only if the resultant union graph G_{1-m} is connected.

Consider an infinite sequence of contiguous, bounded time intervals denoted by $[t_s, t_{s+1})$ for $s = 0, 1, 2, \dots$, with $t_0 = 0$ and a uniform upper bound $t_{s+1} - t_s \leq T$ for some constant $T > 0$. Each interval $[t_s, t_{s+1})$ is partitioned into a finite number m_s of non-overlapping subintervals, $[t_{s,r}, t_{s,r+1})$ for $r = 0, 1, \dots, m_s - 1$, where $t_{s,0} = t_s$ and $t_{s,m_s} = t_{s+1}$. The dwell time on each subinterval is bounded below such that $t_{s,r+1} - t_{s,r} \geq T'$, where $0 < T' < T$. The communication topology is governed by a switching signal $\sigma(t) : [0, \infty) \rightarrow P = \{1, 2, \dots, M\}$, which is piecewise fixed on these subintervals. Here, $M = \lceil T/T' \rceil$ provides an upper bound on the number of possible topology switches within any single interval $[t_s, t_{s+1})$. The graph at time t is denoted $G_{\sigma(t)}$. Assuming all graphs are undirected, the union graph over any interval $[t_s, t_{s+1})$ is also undirected and contains a cycle. This connectivity property ensures persistent information flow and provides redundant communication paths, guaranteeing that the loss of any single link does not isolate any agent.

The ensemble of possible communication topologies is denoted by $\Omega = \{G_1, G_2, \dots, G_m\}$. When subjected to a DoS attack, a subset of communication links may be compromised. The resulting degraded topologies are contained within the set $\Phi = \{\bar{G}_1, \bar{G}_2, \dots, \bar{G}_m\}$, where each $\bar{G}_k \subset G_k$ represents the graph remaining after the attack has dropped a number of edges. The attack is characterized by its maximum duration, T_{DoS} , and its maximum frequency, F_{DoS} , defined as the greatest number of attacks within any interval $[t_s, t_{s+1})$.

During any subinterval $[t_{s,r}, t_{s,r+1})$, an edge may be independently dropped with probability $p \in [0, 1]$. A higher value of p induces greater network degradation. However, the union graph over the interval $[t_s, t_{s+1})$ is assumed to remain undirected and connected (containing a cycle), ensuring persistent information flow.

To maintain network stability, which depends on a minimum dwell time T' for each topology, the attack duration per subinterval must be bounded. This is formalized by the constraint $T_{DoS} \leq \eta T'$, where $\eta \in (0, 1)$ is an attack rate parameter [34]. A larger η permits longer attack. Furthermore, reflecting the adversary's limited resources, the cumulative attack time within any interval $[t_s, t_{s+1})$ must be constrained. The total sleep time is shown as:

$$\tau_f = \sum_{r=0}^{m_s-1} T_{DoS}^{s,r} \leq \eta T \quad (7)$$

Consequently, the number of sustainable attacks per interval is bounded by $N_{DoS} = T/\tau_f$.

To achieve consensus in fixed time, the consensus algorithm proposed in this paper is as follows:

$$\begin{aligned} u_i(t) = l & \sum_{j \in N_i(t)} a_{ij}(t) [\text{sign}(x_j - x_i) |x_j - x_i|^{\alpha_1} \\ & + \text{sign}(x_j - x_i) |x_j - x_i|^{\beta_1} \\ & + \text{sign}(v_j - v_i) |v_j - v_i|^{\alpha_2} |x_j - x_i|^{\alpha_3} \\ & + \text{sign}(v_j - v_i) |v_j - v_i|^{\beta_2}] \\ & + k \sum_{j \in N_i(t)} a_{ij}(t) [(x_j - x_i) + (v_j - v_i)] \end{aligned} \quad (8)$$

where $0 < \alpha_1, \alpha_2, \alpha_3 < 1$ and $\beta_1, \beta_2 > 0$ with the relation $\beta_2 = 2\beta_1/(1 + \beta_1)$ and $2(\alpha_1 - \alpha_3) = (1 + \alpha_1)\alpha_2$ and $k, l > 0$ are the control gains. The goal is to design a distributed control rule u_i that guarantees that all agents reach consensus in both position and velocity states in a finite time, i.e., $x_i(t) \rightarrow x^*$ and $v_i(t) \rightarrow v^*$ for all i in a finite time $T < \infty$. In the proposed protocol, $\text{sign}(z)$ is the sign function, which guarantees that the state of each agent tends towards the state of its neighbors. Also, $a_{ij}(t)$ denotes the element of the time-varying adjacency matrix associated with the communication graph $G_{\sigma(t)}$ and $N_i(t)$ represent the set of neighbors of agent i at time t . The communication topology switches at discrete instants $t_{s,r}$. The set of all possible graphs is finite and given by Ω, Φ . A fundamental assumption is that the union graph (or community graph) over any interval $[t_s, t_{s+1})$, for $s = 0, 1, 2, \dots$, remains connected. Furthermore, the edge weights are fixed over time; if a communication link exists between agents i and j , ($j \in N_i(t)$), its weight is a positive constant $a_{ij}(t) > 0$. Under these conditions, the system dynamics from (4) can be expressed over any interval $[t_s, t_{s+1})$ as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = l \sum_{j \in N_i(t)} a_{ij}(t) [\text{sign}(x_j - x_i) |x_j - x_i|^{\alpha_1} \\ + \text{sign}(x_j - x_i) |x_j - x_i|^{\beta_1} \\ + \text{sign}(v_j - v_i) |v_j - v_i|^{\alpha_2} |x_j - x_i|^{\alpha_3} \\ + \text{sign}(v_j - v_i) |v_j - v_i|^{\beta_2} \\ + k \sum_{j \in N_i(t)} a_{ij}(t) [(x_j - x_i) + (v_j - v_i)] \end{cases} \quad (9)$$

where the weight of the neighborhood matrix $a_{ij}(t) = a_{ij}^{s,r}$ is a constant value at each $t \in [t_{s,r}, t_{s,r+1})$, for $r = 0, 1, \dots, m - 1$.

Theorem 1: Let G be an undirected and jointly-connected variable communication graph. Then, algorithm (8) solves the fixed-time consensus problem for system (4) with the stated conditions under the switching topology and link failures.

3-1- Proof

The system considered as (4) has the state $X_i = [x_i, v_i]^T \in R^{2n}$. Without loss of generality in the consensus analysis, $m = 1$ is assumed in the following, and it can be generalized to any dimension using the Kronecker multiplication.

According to the above, there is a sequence of non-overlapping subintervals $[t_{s,0}, t_{s,1}), [t_{s,1}, t_{s,2}), \dots, [t_{s,m_s-1}, t_{s,m_s})$ in each time interval $[t_s, t_{s+1})$ with the conditions $t_{s+1} - t_s \leq T$, $t_0 = 0$, $t_{s,0} = t_s$ and $t_{s,m_s} = t_{s+1}$ where $m_s \geq 0$ and $T' > 0$ holds in the relation $t_{s,r+1} - t_{s,r} \geq T'$ for $r = 0, 1, \dots, m_s - 1$, so that the communication network G_{σ} changes in $t_{s,r}$ and remains fixed during each subinterval $[t_{s,r}, t_{s,r+1})$. Also, in each time interval $[t_s, t_{s+1})$ there are at most $M = \lceil T/T' \rceil$ subintervals. Given that the graph is undirected, for each

$i, j \in \{1, 2, \dots, n\}$ the relation $a_{ij} = a_{ji}$ holds. In this case, the relation $\sum_{i=1}^n u_i(t) = 0$ holds, which, according to Lemma 1, yields $x^* = (1/n) \sum_{i=1}^n x_i(t)$ and $v^* = (1/n) \sum_{i=1}^n v_i(t)$ which are constant over time. According to the above, the disagreement vector is defined as follows:

$$\begin{cases} \dot{e}_{x_i} = x_i - x^* \\ \dot{e}_{v_i} = v_i - v^* \end{cases}, \quad i = 1, 2, \dots, n \quad (10)$$

by derivation of equation (10), we have:

$$\begin{cases} \dot{e}_{x_i} = e_{v_i} \\ \dot{e}_{v_i} = u_i \end{cases}, \quad i = 1, 2, \dots, n \quad (11)$$

In each interval $[t_s, t_{s+1})$ for $s = 0, 1, 2, \dots$, the Lyapunov function is considered as follows:

$$V = l \sum_{i=1}^n \sum_{j=1}^n \int_0^{e_{x_i} - e_{x_j}} a_{ij}(t) [\text{sign}(z)|z|^{\alpha_1} + \text{sign}(z)|z|^{\beta_1}] dz + \frac{1}{2} e_v^T e_v + \frac{k}{2} e_x^T L_\sigma e_x \quad (12)$$

by derivation of equation (12), we have:

$$\begin{aligned} \dot{V} &= l \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) e_{v_i} \left[\text{sign}(e_{x_i} - e_{x_j}) |e_{x_i} - e_{x_j}|^{\alpha_1} \right. \\ &\quad \left. + \text{sign}(e_{x_i} - e_{x_j}) |e_{x_i} - e_{x_j}|^{\beta_1} \right] \\ &\quad + \sum_{i=1}^n e_{v_i} \dot{e}_{v_i} + k e_x^T L_\sigma e_v \\ &= l \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) e_{v_i} \left[\text{sign}(e_{x_i} - e_{x_j}) |e_{x_i} - e_{x_j}|^{\alpha_1} \right. \\ &\quad \left. + \text{sign}(e_{x_i} - e_{x_j}) |e_{x_i} - e_{x_j}|^{\beta_1} \right] \\ &\quad + \sum_{i=1}^n e_{v_i} \left[l \sum_{j=1}^n a_{ij} \left[\text{sign}(e_{x_j} - e_{x_i}) |e_{x_j} - e_{x_i}|^{\alpha_1} \right. \right. \\ &\quad \left. \left. + \text{sign}(e_{x_j} - e_{x_i}) |e_{x_j} - e_{x_i}|^{\beta_1} \right] \right. \\ &\quad \left. + \text{sign}(e_{v_j} - e_{v_i}) |e_{v_j} - e_{v_i}|^{\alpha_2} |e_{x_j} - e_{x_i}|^{\alpha_3} \right. \\ &\quad \left. + \text{sign}(e_{v_j} - e_{v_i}) |e_{v_j} - e_{v_i}|^{\beta_2} \right] \\ &\quad \left. + k \sum_{j=1}^n a_{ij}(t) \left[(e_{x_j} - e_{x_i}) + (e_{v_j} - e_{v_i}) \right] \right] \\ &\quad + k e_x^T L_\sigma e_v \\ &= l \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) e_{v_i} \left[\text{sign}(e_{v_j} - e_{v_i}) |e_{v_j} - e_{v_i}|^{\beta_2} \right. \\ &\quad \left. + \text{sign}(e_{v_j} - e_{v_i}) |e_{v_j} - e_{v_i}|^{\alpha_2} |e_{x_j} - e_{x_i}|^{\alpha_3} \right] \\ &\quad + k \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) e_{v_i} \left[(e_{x_j} - e_{x_i}) + (e_{v_j} - e_{v_i}) \right] \\ &\quad + k e_x^T L_\sigma e_v \\ &= \frac{1}{2} l \sum_{i=1}^n \sum_{j=1}^n (a_{ij}(t) + a_{ji}(t)) e_{v_i} \\ &\quad \times \left[\text{sign}(e_{v_j} - e_{v_i}) |e_{v_j} - e_{v_i}|^{\beta_2} \right. \\ &\quad \left. + \text{sign}(e_{v_j} - e_{v_i}) |e_{v_j} - e_{v_i}|^{\alpha_2} |e_{x_j} - e_{x_i}|^{\alpha_3} \right] \\ &\quad + \frac{1}{2} k \sum_{i=1}^n \sum_{j=1}^n (a_{ij}(t) + a_{ji}(t)) e_{v_i} \left[(e_{x_j} - e_{x_i}) \right. \\ &\quad \left. + (e_{v_j} - e_{v_i}) \right] + k e_x^T L_\sigma e_v \\ &= \frac{1}{2} l \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) (e_{v_i} - e_{v_j}) \\ &\quad \times \left[\text{sign}(e_{v_j} - e_{v_i}) |e_{v_j} - e_{v_i}|^{\beta_2} \right. \\ &\quad \left. + \text{sign}(e_{v_j} - e_{v_i}) |e_{v_j} - e_{v_i}|^{\alpha_2} |e_{x_j} - e_{x_i}|^{\alpha_3} \right] \end{aligned} \quad (13)$$

$$\begin{aligned} &+ \frac{1}{2} k \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) (e_{v_i} - e_{v_j}) \left[(e_{x_j} - e_{x_i}) \right. \\ &\quad \left. + (e_{v_j} - e_{v_i}) \right] + k e_x^T L_\sigma e_v \\ &= -\frac{1}{2} l \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) \left[|e_{v_j} - e_{v_i}|^{\alpha_2+1} |e_{x_j} - e_{x_i}|^{\alpha_3} \right. \\ &\quad \left. + |e_{v_j} - e_{v_i}|^{\beta_2+1} \right] - \frac{k}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) \left[(e_{v_j} - e_{v_i})^2 \right. \\ &\quad \left. + (e_{v_j} - e_{v_i})(e_{x_j} - e_{x_i}) \right] + k e_x^T L_\sigma e_v \\ &\text{using the relation } x^T Ly = \sum y_i \sum a_{ij}(x_i - x_j) = (1/2) \sum \sum a_{ij}(x_i - x_j)(y_i - y_j) \text{ and the relation } 2a^T P b \leq a^T P a + b^T P b, \text{ for } \dot{V} \text{ we have:} \\ &\dot{V} = -\frac{1}{2} l \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) \left[|e_{v_j} - e_{v_i}|^{\alpha_2+1} |e_{x_j} - e_{x_i}|^{\alpha_3} \right. \\ &\quad \left. + |e_{v_j} - e_{v_i}|^{\beta_2+1} \right] - k e_x^T L_\sigma e_v \\ &\quad - k e_v^T L_\sigma e_v + k e_x^T L_\sigma e_v \quad (14) \\ &= -\frac{1}{2} l \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) \left[|e_{v_j} - e_{v_i}|^{\alpha_2+1} |e_{x_j} - e_{x_i}|^{\alpha_3} \right. \\ &\quad \left. + |e_{v_j} - e_{v_i}|^{\beta_2+1} \right] - k e_v^T L_\sigma e_v \leq 0 \end{aligned}$$

According to equation (14), $\dot{V} < 0$ always holds. From Lemma 2 is the invariant set is considered as $S = \{(x_1, v_1, \dots, x_n, v_n) | \dot{V} = 0\}$. Given that the graph is undirected and connected, it can be seen from equation (14) that $\dot{V} \equiv 0$ when, for each $i \neq j$, $e_{v_i} \equiv e_{v_j}$, which shows that $u_i(t) = u_j(t)$. Therefore, from equation (6) it follows:

$$u_i(t) = l \sum_{j=1}^n a_{ij}(t) \left[\text{sign}(e_{x_j} - e_{x_i}) |e_{x_j} - e_{x_i}|^{\alpha_1} \right. \\ \left. + \text{sign}(e_{x_j} - e_{x_i}) |e_{x_j} - e_{x_i}|^{\beta_1} + k(e_{x_j} - e_{x_i}) \right] \quad (15)$$

given that $a_{ij} = a_{ji}$, we can conclude that $\sum_{i=1}^n u_i = 0$, which implies $u_i \equiv 0$:

$$l \sum_{j=1}^n a_{ij}(t) \left[\text{sign}(e_{x_j} - e_{x_i}) |e_{x_j} - e_{x_i}|^{\alpha_1} \right. \\ \left. + \text{sign}(e_{x_j} - e_{x_i}) |e_{x_j} - e_{x_i}|^{\beta_1} + k(e_{x_j} - e_{x_i}) \right] \equiv 0 \quad (16)$$

like the previous relations, we can write:

$$\begin{aligned} &l \sum_{i=1}^n e_{x_i} \sum_{j=1}^n a_{ij}(t) \left[\text{sign}(e_{x_j} - e_{x_i}) |e_{x_j} - e_{x_i}|^{\alpha_1} \right. \\ &\quad \left. + \text{sign}(e_{x_j} - e_{x_i}) |e_{x_j} - e_{x_i}|^{\beta_1} + k(e_{x_j} - e_{x_i}) \right] \\ &= -\frac{1}{2} l \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) \left[|e_{x_j} - e_{x_i}|^{\alpha_1+1} \right. \\ &\quad \left. + |e_{x_j} - e_{x_i}|^{\beta_1+1} + k(e_{x_j} - e_{x_i})^2 \right] = 0 \end{aligned} \quad (17)$$

Since the topology is connected in every interval $[t_{s_r}, t_{s_r+1})$ for $r = 0, 1, \dots, m_s - 1$ and the graph contains a cycle, so for every agent $i \in \{1, 2, \dots, n\}$, there is at least one $r \in \{0, 1, \dots, m_s - 1\}$ and one agent $j \in \{1, 2, \dots, n\}$ such that $a_{ij}(t) > 0$. So, when t tends to infinity, $e_{x_i} - e_{x_j} \rightarrow 0$ and $e_{v_i} - e_{v_j} \rightarrow 0$ hold, which yields the expressions $x_i - x_j \rightarrow 0$ and $v_i - v_j \rightarrow 0$, and the expressions $x_i = x_j \equiv v^* t + x^*$ and $v_i = v_j \equiv v^*$ are obtained.

The system (4) with variables $[x_1, \dots, x_n, v_1, \dots, v_n]^T$ has the degree of homogeneity $\sigma_1 = \alpha_1 - 1 < 0$ with the dilation $(2, \dots, 2, 1 + \alpha_1, \dots, 1 + \alpha_1)$ and the degree of homogeneity $\sigma_2 = \beta_1 - 1 > 0$ with the dilation $(2, \dots, 2, 1 + \beta_1, \dots, 1 + \beta_1)$ because the system (11) is asymptotically stable and bi-limit homogeneous. Therefore, by Lemma 3, the system is globally fixed-time stable and consensus can be achieved in fixed time. ■

4- Numerical Simulations

In this section, two examples are performed for simulation.

Example 1 (Fixed-time consensus for 10 agents for two scenario):

In example 1, we consider a system with $n = 10$ agents, described by the undirected graphs G_{1-4} shown below:

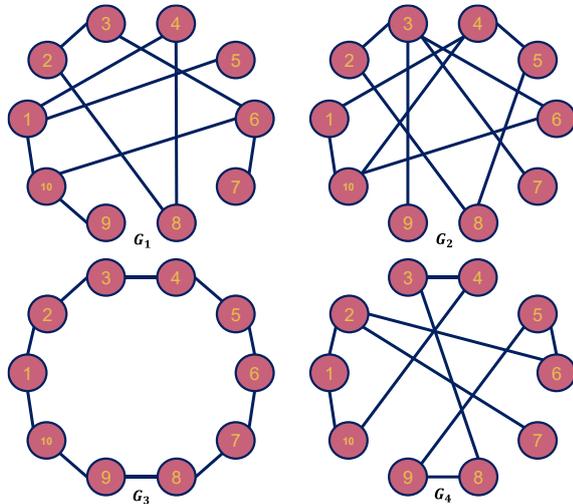


Fig. 2 Undirected and connected graphs to describe 10 agents.

In the simulation, $\alpha_1 = 0.7$, $\alpha_2 = 0.6$, $\beta_1 = 1.2$ and consequently, using the obtained relations for the degree of homogeneity of the system, $\alpha_3 = 0.19$ and $\beta_2 = 1.0909$ are selected. The control gains are considered as $k = l = 1$. The initial conditions are chosen as follows:

Table 1 Initial conditions of the agents

	1	2	3	4	5	6	7	8	9	10
x	0	1	3	2	5	-3	4	-5	-1	-2
v	-1	0	-2	1	4	-5	-3	3	2	-4

At each time step, the graph is randomly selected from the four networks shown in Fig. 2. At every $T = 1$ s, the communication network is randomly changed. In the first scenario, this change is done by selecting one of the four predefined networks to satisfy the conditions assumed in Section 3. Then, at each network change interval, an edge-based DoS attack is applied to the network with a probability of 50%. In the event of an attack, the system is subjected to this attack for a duration of $T_{DoS} = 0.5$ s, Fig. 3. It is assumed that when the attack occurs, the system randomly disconnects some of the communication links to simulate the destructive effects of the attack. The probability of each connection being disconnected is assumed to be 30%, but this disconnection is done in such a way that the communication network remains undirected and connected so that the conditions of Theorem 1 hold. Then, in the second scenario, as shown in Fig. 4, the change period in the communication network is considered to be $T = 0.5$ s and the attack duration is considered to be $T_{DoS} = 0.2$ s to examine the effect of changing the period on the performance of the proposed controller.

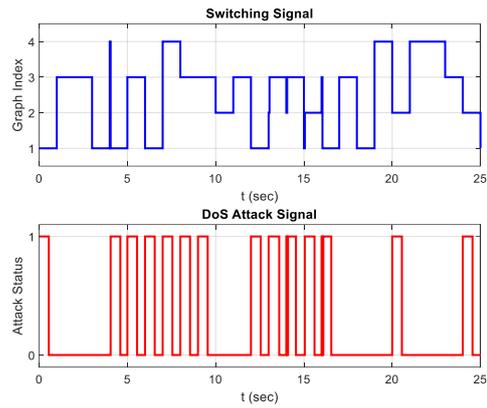


Fig. 3 Switching topology signal and attack signal in the first scenario.

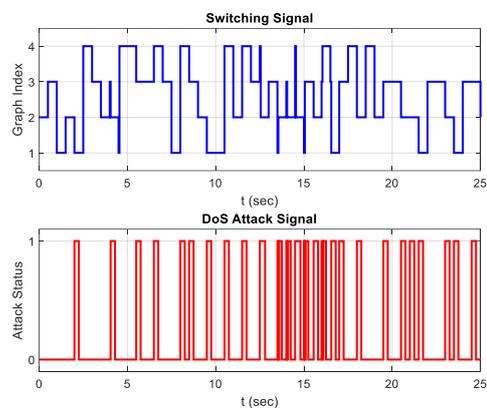


Fig. 4 Switching topology signal and attack signal in the second scenario.

The numerical results presented in Fig. 5 and Fig. 6 demonstrate the effectiveness of the proposed consensus control for the agents' positions. According to Lemma 1 and the description presented in section 3, the agents first converge to the average initial position of the agents and then continue to move at a constant velocity that is equal to the average initial velocity of the agents. Also, according to Fig. 7 and Fig. 8, it can be seen that the velocity of the agents at the time of reaching consensus is equal to the average initial velocity of the agents.

According to Fig. 9 and Fig. 10, it can be seen that the control input of the agents in both scenarios has become zero in a finite time. Finally, according to the above results, it is evident that the system states have reached consensus in a finite time, which confirms the validity of Theorem 1. Also, as shown, as the periodicity of the topology change increases, the convergence speed decreases.

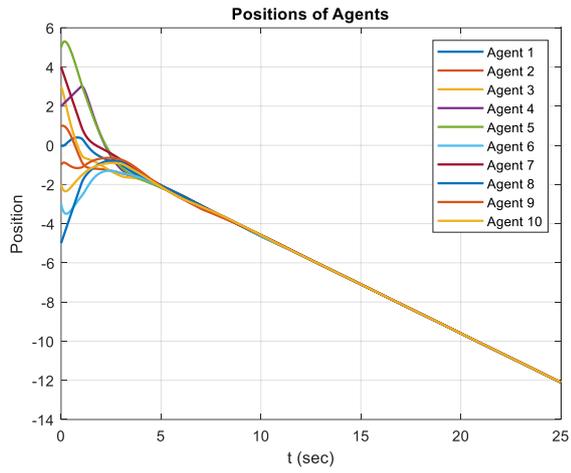


Fig. 5 The position of the agents in the first scenario.

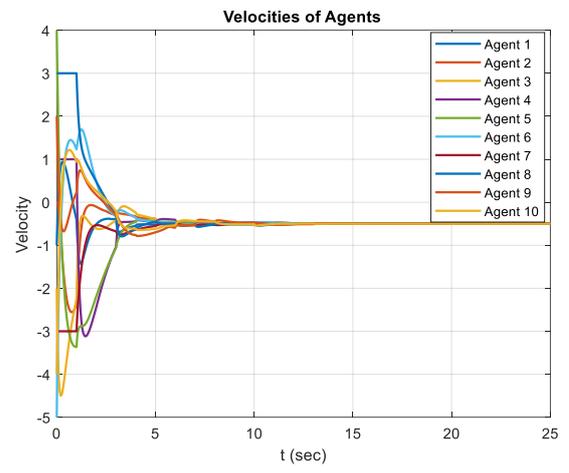


Fig. 7 The velocity of the agents in the first scenario.

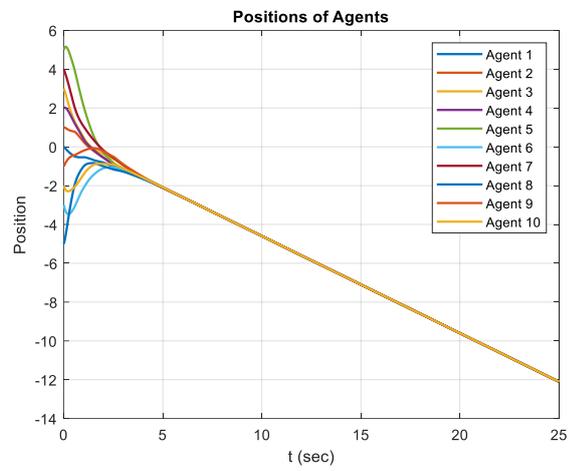


Fig. 6 The position of the agents in the second scenario.

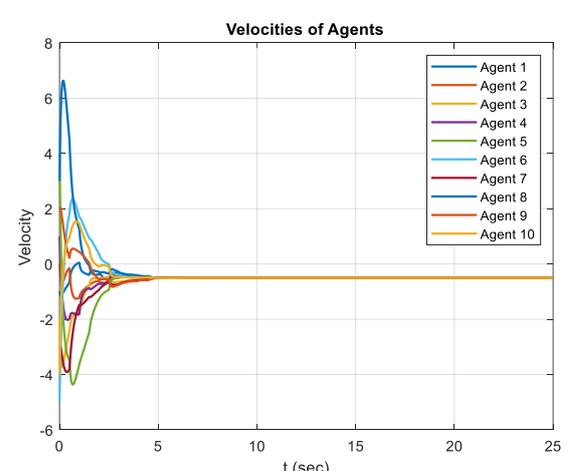


Fig. 8 The velocity of the agents in the second scenario.

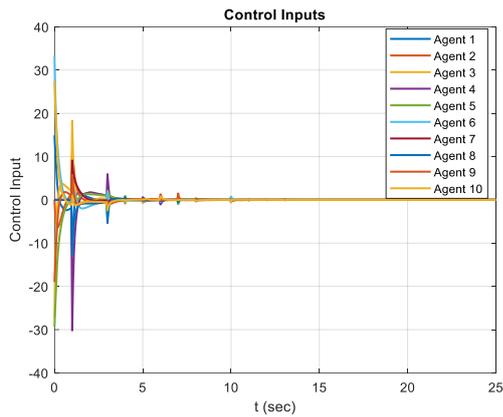


Fig. 9 The control input of the agents in the first scenario.

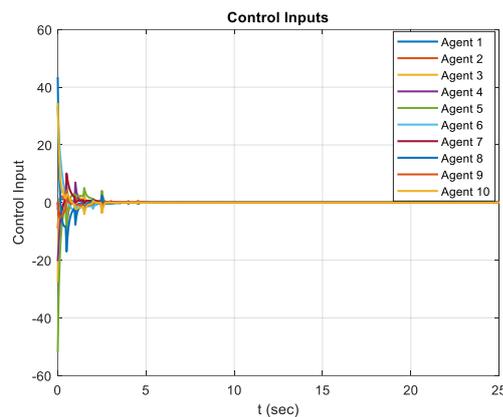


Fig. 10 The control input of the agents in the second scenario.

Example 2 (Fixed-time consensus for 6 agents):

In example 2, we consider a system with $n = 6$ agents, described by the undirected graphs G_{1-6} shown below:

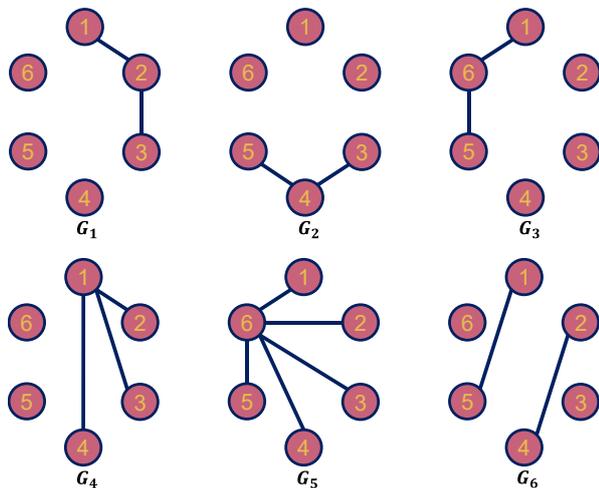


Fig. 11 Undirected and connected graphs to describe 6 agents.

In this example, $\alpha_1 = 0.65$, $\alpha_2 = 0.55$, $\beta_1 = 1.3$ and consequently, using the obtained relations for the degree of homogeneity of the system, $\alpha_3 = 0.1963$ and $\beta_2 = 1.1304$ are selected. The control gains are considered as $k = 1.2$, $l = 0.8$. The initial conditions are chosen as follows:

Table 2 Initial conditions of the agents

	1	2	3	4	5	6
x	1	-3	2	4	5	-4
v	2	0	-1	-2	1	-3

In this case, the controller is assessed under more intricate network dynamics. The attack probability per switching interval is raised to 60% and the topology switching period is shortened to $T = 0.25$ s in order to replicate a highly dynamic environment. As required by Theorem 1, up to 40% of the active communication links are randomly disconnected during an attack that lasts for $T_{DoS} = 0.3$ s while maintaining the undirectedness and connectivity of the graph. Fig. 12 displays the attack and switching signals.

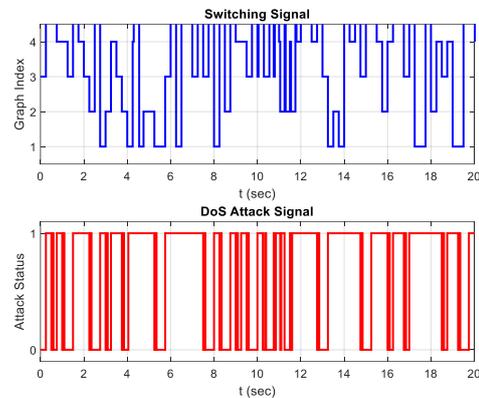


Fig. 12 Switching topology signal and attack signal in the example 2.

The suggested controller successfully brings all agents to consensus in spite of these more difficult circumstances, which are marked by four times faster topology changes and a 20% higher attack probability than in Simulation Case 1. The positions of the agents converge to a common trajectory, as seen in Fig. 13. According to Lemma 1, Fig. 14 verifies that their velocities likewise reach agreement and stabilize at the average of their initial velocities. The control inputs converge to zero in finite time, as shown in Fig. 15. The outcomes show how resilient the controller is and how it guarantees fixed-time convergence even in the face of abrupt and disruptive network changes.

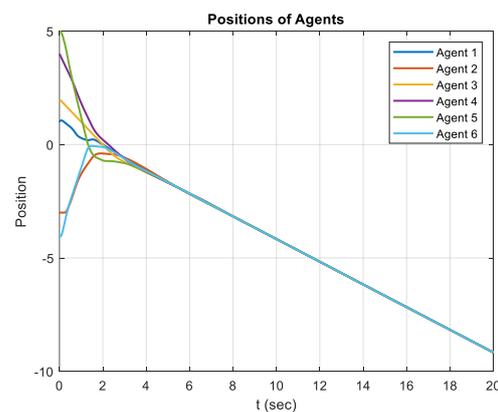


Fig. 13 The position of the 6 agents in the example 2.

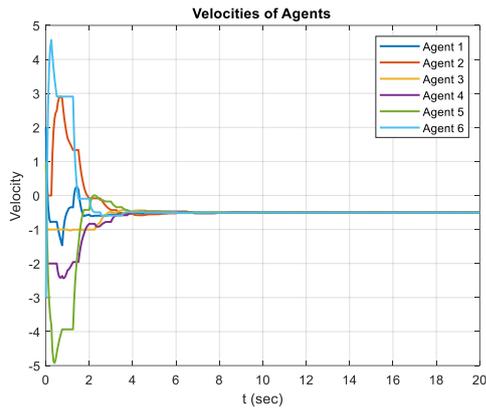


Fig. 14 The velocity of the 6 agents in the example 2.

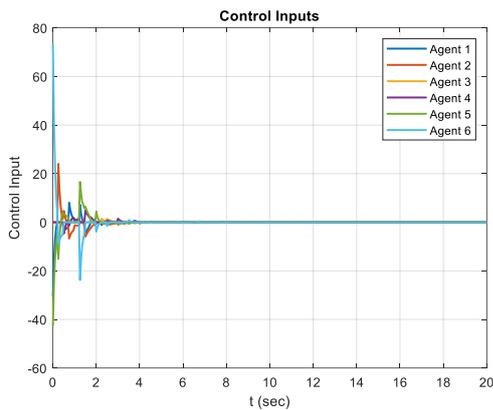


Fig. 15 The control input of the 6 agents in the example 2.

Simulation results show that the proposed algorithm not only achieves consensus under difficult network conditions, but also, due to its fixed-time structure, it is possible to adjust the parameters to achieve optimal and even shorter convergence times in practical applications (such as fast communication networks or fast-response robotic systems).

5- Conclusion

In this paper, the problem of leaderless consensus control for second-order linear MASs under a switching topology with link failure is investigated. To address this problem, an improved fixed-time distributed controller for second-order consensus in MASs with velocity and position coupling is proposed, which guarantees convergence in a finite time. Compared with previous studies, the effects of the simultaneous switching topology with link failure on second-order linear MASs are considered, so that the communication graphs are considered as piecewise fixed and undirected graphs. The stability of the proposed controller is proven based on graph theory and Lyapunov's theorem. Finally, the effectiveness of the proposed method is demonstrated through simulations. It is also concluded that the convergence time improves with the decreasing of the topology change period. In future studies, the consensus problem in nonlinear MASs under variable topology in the presence of external disturbances will be investigated.

Ethics Approval

The scientific content of this paper is the result of the authors' research and has not been published or submitted in any Iranian or international journals.

Conflict of Interest

The authors declared that they have no conflicts of interest to this work.

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